

Neutrinos and Symmetries

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Three facets of symmetries in neutrino physics are briefly reviewed: i) The SO(5) symmetry of the neutrino mass and its connection to the see-saw mechanism; ii) SU(N_{flavor}) symmetries of dense, self-interacting neutrino gases in astrophysical settings; iii) The neutrino mixing angle θ_{13} and possible CP-violation in the neutrino sector.

1. SO(5) SYMMETRY OF THE NEUTRINO MASS

One of the most exciting discoveries in the physics during the last two decades was that neutrinos have mass. This was achieved through the observation of neutrino oscillations. There is no neutrino mass term in the Standard Model. However, using the Standard Model degrees of freedom one can parameterize the neutrino mass by a dimension-five operator: $X_{\alpha\beta} H H \overline{\nu_{L\alpha}^C} \nu_{L\beta} / \Lambda$. This term is not renormalizable. Furthermore it is the only dimension-five operator one can write using the Standard Model degrees of freedom. Hence the neutrino mass is, in some sense, the most accessible new physics beyond the Standard Model. One can also write down another mass term: $\overline{\nu_R^C} \nu_R$, which is permitted by the weak-isospin invariance of the Standard Model. Such Majorana mass terms violate lepton number conservation since they imply that neutrinos are their own antiparticles.

Back in 1957, Pauli and Gürsey considered a particle-antiparticle symmetry, realized via the transformation

$$\Psi \rightarrow a\Psi + b\gamma_5\Psi^C, \quad |a|^2 + |b|^2 = 1. \quad (1)$$

It is easy to see that, under such a transformation, a pure Dirac mass terms would transform into a mixture of Dirac and Majorana mass terms. Indeed it is easy to show that the "charge" (as opposed to "current") operators

$$D_+ = \frac{1}{2} \int d^3\mathbf{x} \overline{\Psi}_L \Psi_R, \quad (2)$$

$$A_+ = \int d^3\mathbf{x} \left[-\Psi_L^T \mathcal{C} \gamma_0 \Psi_R \right], \quad (3)$$

$$L_+ = \frac{1}{2} \int d^3\mathbf{x} \left(\overline{\Psi}_L \Psi_L^C \right), \quad R_+ = \frac{1}{2} \int d^3\mathbf{x} \left(\overline{\Psi}_R^C \Psi_R \right), \quad (4)$$

their complex conjugates, and the operators

$$L_0 = \frac{1}{4} \int d^3\mathbf{x} \left(\Psi_L^\dagger \Psi_L - \Psi_L \Psi_L^\dagger \right), \quad R_0 = \frac{1}{4} \int d^3\mathbf{x} \left(\Psi_R \Psi_R^\dagger - \Psi_R^\dagger \Psi_R \right) \quad (5)$$

form an $SO(5)$ algebra [2]. The operators A_+, A_- and $A_0 = R_0 - L_0$ form an $SU(2)$ subalgebra that generates the Pauli-Gürsey transformation ($SU(2)_{PG}$). The most general neutrino mass Hamiltonian sits in the $SO(5)/SU(2)_L \times SU(2)_R \times U(1)_{L_0+R_0}$ coset and can be diagonalized by a $SU(2)_{PG}$ rotation, producing the see-saw masses. This $SO(5)$ is the largest symmetry associated with neutrino mass terms and its implications are not yet fully explored.

2. FLAVOR SYMMETRIES OF DENSE NEUTRINO GASES

Dense, self-interacting neutrino gases, encountered in astrophysical settings such as core-collapse supernovae, possess an interesting, non-linear $SU(N_{\text{flavor}})$ symmetry. The standard MSW potential is provided by the coherent forward scattering of ν_e 's off the electrons in dense matter via W-exchange. There is a similar term with Z-exchange. But since it is the same for all neutrino flavors, it does not contribute at the tree level to phase differences that drive the MSW effect unless we invoke a sterile neutrino. Neutrinos play a salient role in the dynamics of core-collapse supernovae and hence it is crucial to explore all aspects of supernova neutrino physics [3]. If the neutrino density itself is also very high then one has to consider the effects of neutrinos scattering off other neutrinos; this is the case for a core-collapse supernova.

For simplicity, let us consider only two flavors of neutrinos: electron neutrino, ν_e , and another flavor, ν_x . Introducing the creation and annihilation operators for one neutrino with three momentum \mathbf{p} , we can write down the generators of an $SU(2)$ algebra [4]:

$$\begin{aligned} J_+(\mathbf{p}) &= a_x^\dagger(\mathbf{p})a_e(\mathbf{p}), \quad J_-(\mathbf{p}) = a_e^\dagger(\mathbf{p})a_x(\mathbf{p}), \\ J_0(\mathbf{p}) &= \frac{1}{2} \left(a_x^\dagger(\mathbf{p})a_x(\mathbf{p}) - a_e^\dagger(\mathbf{p})a_e(\mathbf{p}) \right). \end{aligned} \quad (6)$$

Note that the integrals of these operators over all possible values of momenta also generate a global $SU(2)$ algebra. Using the operators in Eq. (6) the Hamiltonian for a neutrino propagating through matter takes the form

$$H_\nu = \int d^3\mathbf{p} \frac{\delta m^2}{2p} \left[\cos 2\theta J_0(\mathbf{p}) + \frac{1}{2} \sin 2\theta (J_+(\mathbf{p}) + J_-(\mathbf{p})) \right] - \sqrt{2}G_F \int d^3\mathbf{p} N_e J_0(\mathbf{p}). \quad (7)$$

In Eq. (7), the first integral represents the neutrino mixing and the second integral represents the neutrino forward scattering off the background matter. Neutrino-neutrino interactions are described by the Hamiltonian

$$H_{\nu\nu} = \sqrt{2} \frac{G_F}{V} \int d^3\mathbf{p} d^3\mathbf{q} (1 - \cos \vartheta_{\mathbf{pq}}) \mathbf{J}(\mathbf{p}) \cdot \mathbf{J}(\mathbf{q}), \quad (8)$$

where $\vartheta_{\mathbf{pq}}$ is the angle between neutrino momenta \mathbf{p} and \mathbf{q} and V is the normalization volume. Note that the $(1 - \cos \vartheta_{\mathbf{pq}})$ term in the integral above comes from the V-A nature of the weak interactions and its presence is crucial to recover the effects of the weak interaction physics in the most general situation. Note that in the extremely idealized case of isotropic neutrino distribution and a very large number of neutrinos this term may average to a constant¹ and the neutrino-neutrino interaction Hamiltonian simply reduces

¹Although the number of neutrinos in a core-collapse supernova is very large ($\sim 10^{58}$), their distribution is very unlikely to be isotropic.

to the Casimir operator of the global $SU(2)$ algebra. Inclusion of antineutrinos in Eqs. (7) and (8) introduces a second set of $SU(2)$ algebras. For three flavors one needs two sets of $SU(3)$ algebras, one for neutrinos and one for antineutrinos [5].

Exact solutions of the combined Hamiltonian (Eqs. (7) and (8)) seem to be very difficult to obtain, but a saddle-point approximation [4] to its solutions, where neutrinos interact with a *neutrino* mean-field, is widely used in the literature to numerically investigate the underlying nonlinear behavior [6]. A recent comprehensive review is given in Ref. [7].

Unlike the typical many-body systems in condensed-matter physics (which are mostly controlled by electromagnetism) or in nuclear physics (which are mostly controlled by the strong force), this neutrino gas is the only example of a non-trivial many-body system entirely controlled by the weak interactions. This example illustrates that astrophysical extremes allow testing neutrino properties in ways that cannot be done elsewhere, e.g. $\nu - \nu$ effect as an emergent phenomenon.

3. CP-VIOLATION AND NEUTRINO PHYSICS

We now turn to the fundamental symmetry CP and its possible violation in the neutrino sector. The neutrino mixing matrix is

$$\mathbf{T}_{23}\mathbf{T}_{13}\mathbf{T}_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -S_{13}e^{i\delta} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (9)$$

where $C_{ij} = \cos \theta_{ij}$, $S_{ij} = \sin \theta_{ij}$, and δ is the CP-violating phase. Clearly a non-zero value of θ_{13} would also make the observation of the effects that depend on the CP-violating phase possible. There already are hints for a non-zero value of θ_{13} from solar, atmospheric, and reactor data [8, 9] that will be probed by the reactor experiments in the near future [10, 11, 12].

It would be interesting to explore if matter amplifies or suppresses CP-violating effects. To this end introducing the operators [13]

$$\tilde{\Psi}_\mu = \cos \theta_{23} \Psi_\mu - \sin \theta_{23} \Psi_\tau,$$

$$\tilde{\Psi}_\tau = \sin \theta_{23} \Psi_\mu + \cos \theta_{23} \Psi_\tau,$$

one can write down the neutrino evolution equations as

$$i \frac{\partial}{\partial t} \begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix} = \tilde{\mathbf{H}} \begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix} \quad (10)$$

where

$$\tilde{\mathbf{H}} = \mathbf{T}_{13}\mathbf{T}_{12} \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \mathbf{T}_{12}^\dagger \mathbf{T}_{13}^\dagger + \begin{pmatrix} V_{e\mu} & 0 & 0 \\ 0 & S_{23}^2 V_{\tau\mu} & -C_{23} S_{23} V_{\tau\mu} \\ 0 & -C_{23} S_{23} V_{\tau\mu} & C_{23}^2 V_{\tau\mu} \end{pmatrix}. \quad (11)$$

In writing Eq. (11) a term proportional to identity is dropped by adding a term to all the matter potentials so that $V_{\mu\mu} = 0$.

If we can neglect the potential $V_{\tau\mu}$ it is straightforward to show that

$$\tilde{H}(\delta) = \mathbf{S}\tilde{H}(\delta = 0)\mathbf{S}^\dagger$$

with

$$\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}.$$

This factorization gives us interesting sum rules: Electron neutrino survival probability, $P(\nu_e \rightarrow \nu_e)$ is independent of the value of the CP-violating phase, δ ; or equivalently the combination $P(\nu_\mu \rightarrow \nu_e) + P(\nu_\tau \rightarrow \nu_e)$ at a fixed energy is independent of the value of the CP-violating phase [14]. It is possible to derive similar sum rules for other amplitudes [15]. These results hold even if the neutrino-neutrino interactions are included in the Hamiltonian [16]. Electron neutrino flux at a distance r from the neutrinosphere is

$$\mathcal{L}_e^{(r)} = \mathcal{L}_e^{(0)} P(\nu_e \rightarrow \nu_e, r) + \mathcal{L}_\mu^{(0)} P(\nu_\mu \rightarrow \nu_e, r) + \mathcal{L}_\tau^{(0)} P(\nu_\tau \rightarrow \nu_e, r).$$

If the ν_μ and ν_τ luminosities are the same at the neutrinosphere, i.e. $\mathcal{L}_\mu^{(0)} = \mathcal{L}_\tau^{(0)}$, we get

$$\mathcal{L}_e^{(r)} = \mathcal{L}_e^{(0)} \{P(\nu_e \rightarrow \nu_e, r)\} + \mathcal{L}_\mu^{(0)} \{P(\nu_\mu \rightarrow \nu_e, r) + P(\nu_\tau \rightarrow \nu_e, r)\}$$

Since in the factorizable limit the quantities inside the curly brackets do not depend on the CP-violating phase, δ , we conclude that

$$\mathcal{L}_e^{(r)}(\delta \neq 0) = \mathcal{L}_e^{(r)}(\delta = 0),$$

i.e. under the assumptions stated above, electron neutrino survival probability and, consequently, electron neutrino and antineutrino luminosities are independent of the CP-violating phase. To be able to observe the effects of δ , we need to relax the underlying assumptions by either i) Permitting the ν_μ and ν_τ luminosities to be different at the neutrinosphere (Standard Model (SM) loop corrections and also physics beyond the Standard Model may do this); or ii) Exploring when $V_{\tau\mu}$ is non-zero due to SM loop corrections or due to physics beyond SM. The impact of such loop corrections on supernova physics was first explored in Ref. [17].

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